The Solow-Swan Model

The Solow-Swan model of economic growth postulates a continuous production function linking output to the inputs of capital and labour which leads to the steady state equilibrium of the economy.

Assumptions:

(1) One composite commodity is produced.
(2) Output is regarded net output after making allowance for the depreciation of capital.
(3) There are constant returns to scale.

(4) There are diminishing returns to an individual input.
(5) The two factors of production - labour and capital - are paid according to their marginal physical productivities.
(6) prices and wages are flexible.
(7) There is perpetual full employment of labour.
(8) There is also full employment of the available stock of capital.
(9) Labour and capital are substitutable for each other.
(10) There is no technical progress.
(11) The saving ratio is constant.
(12) Saving equals investment.
(13) Capital depreciates at constant rate, d.
(14) population grows at a constant rate, n.

The Model

The production function

$$Y = F(K, L)$$

where,

Y = income or output

K = Capital

L = Labour

The condition of constant returns to scale implies that if we divide by L, the production function can be written as

$$Y/L = F(K/L, 1) = L \cdot f(k) \quad \text{——— (1)}$$

where, $y = Y/L$ (income per worker)

$k = K/L$ (capital ~~output~~ labour ratio)

$f(k) = f(k, 1)$

Thus, the production function can be expressed as $y = f(k)$ ——— (2)

In the Solow-Swan model, saving is a constant fraction, s, of income. So saving per worker is sy. Since income equals output,

$$sy = sf(k) \quad \text{——— (3)}$$

The investment required to maintain capital per worker k, depends on population growth, and the depreciation rate, d. Since depreciation is a constant, d, percent of the capital stock, $d \cdot k$ is the investment needed to replace worn-out capital. This depreciation investment per worker $d \cdot k$ is added to $n \cdot k$, the investment per worker to maintain capital-labour ratio for the growing population,

$$(nk + dk) = (n + d)k \quad \text{——— (4)}$$

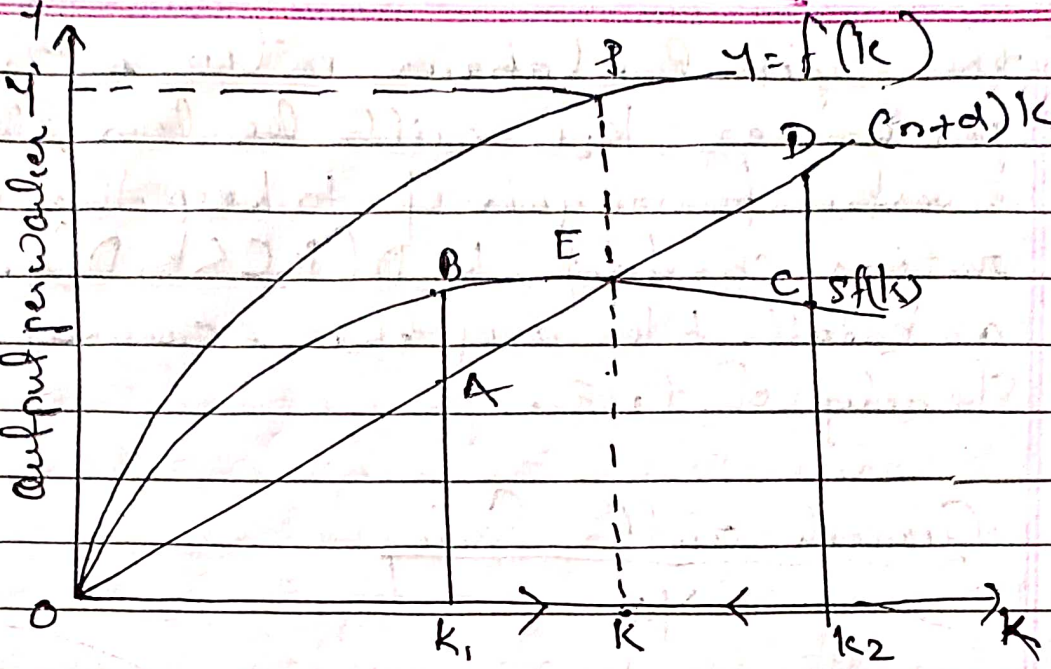
which is the investment required to maintain capital per worker.

The net change is capital per worker (capital labour ratio) k' overtime is the excess of saving per worker over the required investment to maintain capital per worker,

$$k' = sf(k) - (n + d)k \quad \text{——— (5)}$$

This is the fundamental equation for the Solow-Swan model, where the steady state corresponds to $k' = 0$. The economy reaches a steady state when

$$sf(k) = (n + d)k \quad \text{——— (6)}$$



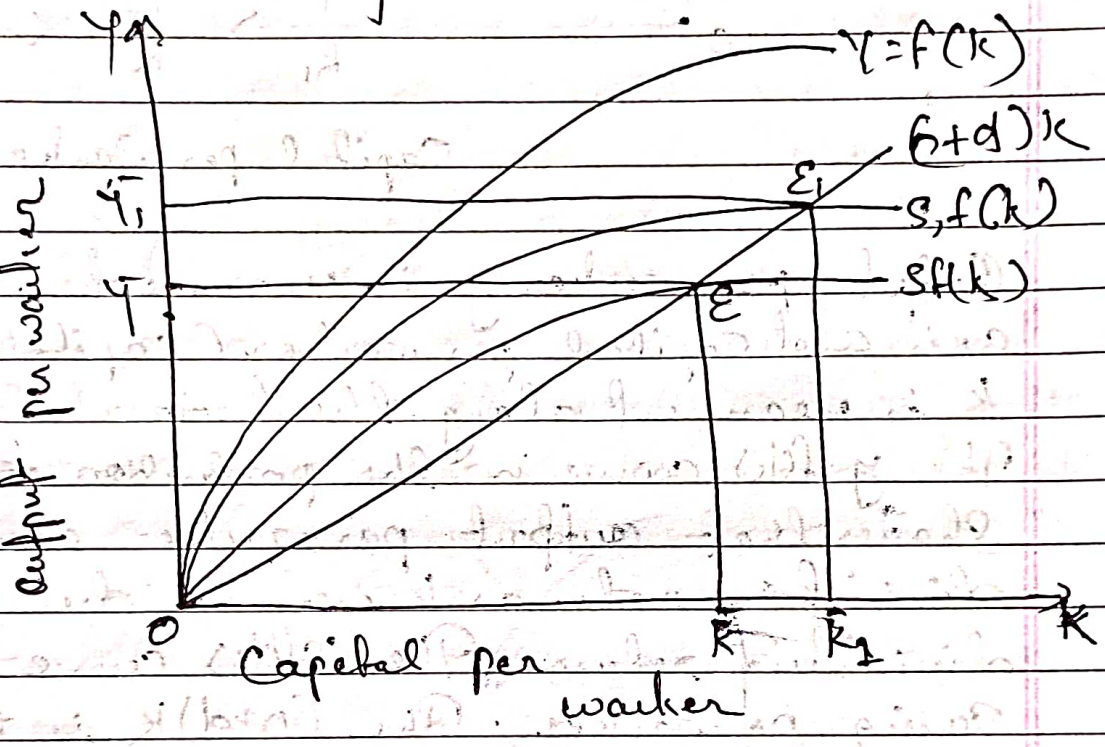
Output per worker y is measured along the vertical axis and capital per worker (capital-labour ratio) k, is measured along the horizontal axis.

The $y=f(k)$ curve is the production function which shows that output per worker increases at a diminishing rate as k increases due to the law of diminishing returns. The $sf(k)$ curve represents saving per worker. The $(n+d)k$ is the investment requirement line from the origin with a positive slope equal to $(n+d)$. The steady state level of capital, k^* is determined where the $sf(k)$ curve intersects the $(n+d)k$ line at point E. The steady state income is y^* with output per worker k^*P, as measured by point P on the production function $y=f(k)$

Suppose the economy starts at the capital-labour ratio k_1. Here saving per worker k_1B exceeds the investment required to keep the capital-labour ratio constant, k_1A ($k_1B > k_1A$). Thus, k and y increase until k^* is reached when the economy is in the steady state at point E. Alternatively, if

the capital - labour ratio is k_2, the saving per worker, k_2C will be less than the investment required to keep the capital-labour ratios constant, k_2D ($k_2C < k_2D$). Thus y will fall as k falls to k' and the economy reaches the steady state E.

Growth with Saving



According to Solow-Swan model is the steady state, both k and y being constant, the growth rate is not affected by the saving rate. In this figure, K is the steady state capital per worker and y output per worker when the $sf(k)$ curve intersects the $(\delta+d)k$ curve at point E. An increase in the saving rate from s to s_1 shifts the saving curve $sf(k)$ upward to $s_1f(k)$. The new steady state point is E_1.

When the saving rate increases from s to s_1 with no change in the growth rate of labour force (n), the capital per worker will continue

to rise to k_1 which will raise output per worker to y_1 and so will the growth rate of output increases. But this process continues at a diminishing rate in the transition period. As a result, the initial growth rate of output is restored over the long run at the new steady state equilibrium point E_1, where $(n+d)k = s_1 f(k)$. After this point, there will be no further increase in output per worker because the growth rate of labour force (n) does not change and the long run growth rate of output also remains at the same level.

Implications

1- The growth rate of output in steady state is exogenous and is independent of the saving rate and of technical progress.

2- If saving rate increase, it increases the output per worker by increasing the capital per worker, but the growth rate of output is not affected.

3- The growth in per capita income can either be achieved by increased saving or reduced rate of population growth. This will hold if depreciation is allowed in the model.

4- Another prediction of the model is that in the absence of continuing improvements in technology, growth per worker must ultimately cease.

5- This model predicts conditional convergence. All countries having smaller characteristics like saving rate, population growth rate, technology etc. that affect growth will converge to the same steady state level. It means the poor countries having the same saving rate and level of technology of the rich countries will reach the same steady growth rates in the long run.

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